NSF-ITP-97-021, hep-th/9703055

Errata for: Differential Equations for Sine-Gordon Correlation Functions at the Free Fermion Point

Denis Bernard[1]

Service de Physique Théorique, CEN-Saclay[2]
F-91191 Gif sur Yvette, France

André Leclair[3]

Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106-4030

We present some important corrections to our work which appeared in Nucl. Phys. B426 (1994) 534. Our previous results for the correlation functions $\langle e^{i\alpha\Phi(x)} e^{i\alpha'\Phi(0)}\rangle$ were only valid for $\alpha = \alpha'$, due to the fact that we didn't find the most general solution to the differential equations we derived. Here we present the solution corresponding to $\alpha \neq \alpha'$.

2/97

---

[1] Member of CNRS

[2] Laboratoire de la Direction des sciences de la matière du Commissariat à l'énergie atomique.

[3] On leave from Cornell University

Appendix E. Errata

In section 3 we did not find the most general solution to the differential equations (3.37) when we imposed $\partial_z a = \partial_z b = \partial_{\overline{z}}(b-a) = 0$. We now understand that for $\alpha \neq \alpha'$, the latter condition is not valid. In this errata we present the modifications for $\alpha \neq \alpha'$. A corrected version of the paper which incorporates the modifications below is available[1][4]

(1) Equation 1.3 should be replaced with:

$$\begin{aligned} \left(\partial_r^2 + \frac{1}{r}\partial_r\right)\Sigma(r) &= \frac{m^2}{2}\left(1-\cosh 2\varphi\right) \\ \left(\partial_r^2 + \frac{1}{r}\partial_r\right)\varphi &= \frac{m^2}{2}\sinh 2\varphi + \frac{4(\alpha-\alpha')^2}{r^2}\tanh\varphi(1-\tanh^2\varphi), \end{aligned} \tag{E.1}$$

where $r^2 = 4z\overline{z}$, and m is the mass....

(2) In equation (3.31), $\partial_{\overline{z}} B_+ = \frac{m}{2}\widehat{C}_+ C_-$ should be replaced with $\partial_{\overline{z}}\widehat{B}_+ = \frac{m}{2}\widehat{C}_+ C_-$.

(3) The end of section 3, beginning with the sentence after (3.38), should be replaced with the following:

Inserting this parameterization into the differential equations gives the following. The first two equations in (3.37) give

$$\partial_z a = -\tanh^2\varphi\ \partial_z b. \tag{E.2}$$

Using this equation and its $\partial_{\overline{z}}$ derivative the second 2 equations can be simplified to

$$\begin{aligned} (\partial_z\partial_{\overline{z}}a)\ \coth\varphi - (\partial_z\partial_{\overline{z}}b)\ \tanh\varphi - 2\partial_z\varphi\ \partial_{\overline{z}}(b-a) &= 0 \\ \partial_z\partial_{\overline{z}}\varphi = \frac{m^2}{2}\sinh 2\varphi - \tanh\varphi\ \partial_z b\ \partial_{\overline{z}}(b-a)&. \end{aligned} \tag{E.3}$$

The function b can be deduced using Lorentz invariance. Let $z = re^{i\theta}/2$, $\overline{z} = re^{-i\theta}/2$, and consider shifts of θ by γ. The functions $e, \widehat{e}$ satisfy

$$\begin{aligned} e(e^{i\gamma}z, e^{-i\gamma}\overline{z}, u) &= e^{-i\gamma(1+\alpha'-\alpha)/2}e(z,\overline{z},e^{i\gamma}u) \\ \widehat{e}(e^{i\gamma}z, e^{-i\gamma}\overline{z}, u) &= e^{-i\gamma(1+\alpha-\alpha')/2}e(z,\overline{z},e^{i\gamma}u). \end{aligned} \tag{E.4}$$

[4] Recently, similar results were obtained using different methods in [2].

From the definition (3.21) of $C_+, \widehat{C_+}$, and making the change of variables $u \to e^{-i\gamma}u$, one finds

$$C_+ = e^{2i(\alpha-\alpha')\theta} f(r), \qquad \widehat{C}_+ = e^{-2i(\alpha-\alpha')\theta} \widehat{f}(r), \tag{E.5}$$

for some scalar functions $f, \widehat{f}$. Then, using

$$e(z, \overline{z}, u) = u\ \widehat{e}(\overline{z}, z, 1/u), \tag{E.6}$$

one can show $f = \widehat{f}$ by making the change of variables $u \to 1/u$. Thus,

$$e^{2b} = \frac{C_+}{\widehat{C}_+} = e^{4i(\alpha-\alpha')\theta}, \tag{E.7}$$

and

$$b = (\alpha - \alpha') \log\left(\frac{z}{\overline{z}}\right). \tag{E.8}$$

Inserting this b into (E.2) and taking the complex conjugate, one deduces

$$\partial_{\overline{z}} a = \tanh^2 \varphi\ \partial_{\overline{z}} b. \tag{E.9}$$

The function φ is only a function of r. Thus (E.3) can be written as

$$\left(\partial_r^2 + \frac{1}{r}\partial_r\right) \varphi = \frac{m^2}{2} \sinh 2\varphi + \frac{4(\alpha-\alpha')^2}{r^2} \tanh \varphi (1 - \tanh^2 \varphi). \tag{E.10}$$

Finally, using the equation (3.36), and also (3.38), one obtains

$$\left(\partial_r^2 + \frac{1}{r}\partial_r\right) \Sigma(r) = -m^2\ \sinh^2 \varphi = \frac{m^2}{2} \left(1 - \cosh 2\varphi\right). \tag{E.11}$$

This is the result announced in the introduction. Notice that $\partial_z \partial_{\overline{z}} \Sigma$ is only parameterized by a single function $\varphi(r)$, and the differential equation for φ involves only φ itself.

We thank S. Lukyanov for first pointing out a possible error in the original paper. The result (E.1) was used in the work [3].

This work is supported by the National Science foundation, in part through the National Young Investigator Program, and under Grant No. PHY94-07194.

References

[1] D. Bernard and A. Leclair, hep-th/9402144.

[2] H. Widom, *An Integral Operator Solution to the Matrix Toda Equations*, solv-int 9702007.

[3] S. Lukyanov and A. Zamolodchikov, *Exact expectation values of local fields in quantum sine-Gordon model*, hep-th/9611238.